THE METHOD OF EFFECTIVE ENERGY IN THE PROBLEM OF A POWERFUL EXPLOSION IN A REAL GAS

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In [1] the author proposed the use of an effective nose resistance coefficient for calculating the effect of the real properties of a gas in a high entropy layer for hypersonic flow around thin blunt bodies. The essence of the method lies in the exclusion of the energy involved in physicochemical transformations from the balance of kinetic and potential energy.

In what follows this method is extended to the case of a strong explosion [2] in a real gas, important from the point of view of the explosion analogy of flow around thin blunt bodies [3]. We represent the equations of motion for this case in the following integral form (γ is the adiabatic exponent of the unperturbed gas):

$$\frac{1}{2}\rho_\infty R^\nu v_R{}^2 I_1 + \frac{p_0 R^\nu}{\gamma-1} I_2 = \frac{\nu}{l} E^* + \frac{R^\nu p_\infty}{(\gamma-1)}, \quad E^* = \mu E,$$
$$\rho_\infty R^\nu v_R I_3 = \nu \int_0^t R^{\nu-1}(p_0 I_4 - p_\infty)\,dt,$$
$$E^* = \mu E, \quad v_R = \dot{R}\left(1 - \frac{\rho_\infty}{\rho_R}\right), \quad \dot{R} = \frac{\partial R}{\partial t},$$
$$I_1 = \frac{1}{M}\int_0^M \left(\frac{v}{v_R}\right)^2 dm, \quad I_2 = \frac{\nu}{R^\nu}\int_0^R \frac{p}{p_0} r^{\nu-1}\,dr,$$
$$I_3 = \frac{1}{M}\int_0^M \left(\frac{v}{v_R}\right) dm,$$
$$I_4 = \frac{\nu-1}{R^{\nu-1}}\int_0^R \frac{p}{p_0} r^{\nu-2}\,dr \quad \text{for} \quad \nu = 2, 3,$$
$$I_4 = 1 \quad \text{for} \quad \nu = 1,$$
$$dm = l\rho r^{\nu-1} dr, \quad M = (l/\nu)\rho_\infty R^\nu,$$
$$l = 1,\ 2\pi,\ 4\pi \quad \text{for} \quad \nu = 1, 2, 3. \tag{1}$$

Here and in what follows ρ, p, e, i, R, and v are the density, pressure, internal energy, enthalpy, gas shock wave and gas velocity; r and t are the distance to the centre and evolution time of the explosion, the indices ∞, R, and 0 refer to quantities in the unperturbed gas, directly behind the shock wave, and in the center; the quantity ν corresponds to the dimensionality of the space; P is the total energy of the explosion; $E^* = \mu E$ is the effective energy.

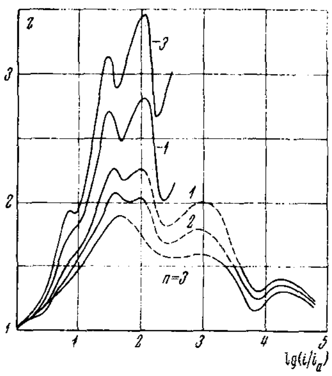

Fig. 1

The coefficient μ takes into account the difference of potential energies between real and perfect gases, and for the equation of state

$$\frac{\rho i}{p} = \frac{\gamma}{\gamma-1} z(i, p) = \frac{\gamma_0}{\gamma_0 - 1} \tag{2}$$

it may be represented in the following form:

$$\mu = 1 - \frac{1}{E}\int_0^M \left(e - \frac{p}{\rho(\gamma-1)}\right) dm =$$
$$= 1 - \int_0^{M_0} \frac{i}{e_\infty}\frac{z-1}{z}\,dm_0 \quad \left(m_0 = \frac{me_\infty}{E}\right). \tag{3}$$

The function z is shown in Fig. 1 for air (γ = 1.4) for various $p = 10^n$ atm. The solid lines are the data of [4] for temperatures $T \leqslant 20\,000^\circ$ K, the dashed lines are the data of [5] for temperatures $T \leqslant 500\,000^\circ$ K, and i_a = 250 cal/g is the enthalpy for T = 1000° K.

For an attenuated shock wave with $R \leqslant 6a_\infty \simeq 2000$ m/sec the air will be dissociated only in the central zone with a practically fixed mass m_0. Since μ is only weakly dependent upon the pressure (as $p^{(\gamma_0-1)/\gamma_0}$) the analysis of paper [1] may be applied in its entirety to this case. Here the mass m_0 is analogous to the high entropy layer; the law of motion of the shock wave, as well as distribution of parameters outside the mass m_0, will coincide at all times with the same quantities for an explosion in a perfect gas with energy $E^* = \mu E$.

Let us now consider a more general case. Usually $(\rho_R/\rho_\infty \approx 6-20$ for powerful shock waves in air, and as is well known from the analysis of the exact solutions [2, 6], the main mass of gas is situated in a narrow region of order (ρ_∞/ρ_R) R in the neighborhood of the shock wave, outside which the pressure is close to a constant. Thus the integrals I_k and the ratio $v_R/\dot{R}$ are close to unity, and consequently are only weakly dependent on the equation of state of the gas. However, since with these assumptions Eqs. (1) completely determine the law of motion of the shock wave R(t) and the pressure $p_0(t)$, the effect of the real properties of the gas on these basic quantities will manifest itself only through the coefficients μ.

The function μ depends on time basically because of the dependence of the function z on the pressure, i.e., it is a comparatively weak relationship (Fig. 1); thus we assume in first approximation that, as for the case of blunt bodies [1], the solution of system (1) is at all times close to that with constant μ, equal to its local value.

Thus the required solution may be given the following simple form:

$$\frac{p_R}{p_\infty} = 1 + \varkappa_\nu(\gamma)\frac{\mu}{M_0} f_1(R^*),$$
$$i_R - i_\infty = \frac{1}{2}(p_R - p_\infty)\left(\frac{1}{\rho_\infty} + \frac{1}{\rho_R}\right), \tag{4}$$
$$R^* = R\left(\frac{p_\infty}{E^*}\right)^{1/\nu} = \chi_\nu(\gamma)(\tau^*)^{2/(\nu+2)} f_2(\tau^*),$$
$$\left(M_0 = \frac{Me_\infty}{E}, \quad \tau^* = t\left(\frac{p_\infty}{\rho_\infty}\right)^{1/2}\left(\frac{p_\infty}{E^*}\right)^{1/\nu},\right.$$
$$\left.\varkappa_1 = 1.75, \quad \varkappa_2 = 1.67, \quad \varkappa_3 = 1.65\right). \tag{5}$$

Here f_1 and f_2 may be found from the exact calculations for a perfect gas (for example, [7]) or from any approximate solutions (for example, [6, 8]). A similar formula may be written down for p_0/p_∞ also.

As distinct from flow around blunt bodies where the form of the front portion is known in advance, the law of motion of a shock wave

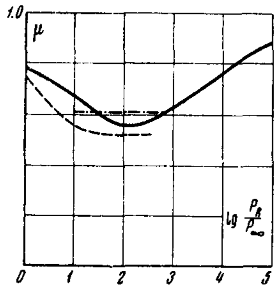

Fig. 2

in an explosion and the entropy distribution in the central part of the explosives zone are determined by a process of simultaneous solution of the relationships (2)-(4), or (1)-(3). The enthalpy profile which is necessary for calculating μ can be determined by successive integration of the adiabatic equation $d\ln i = [(\gamma_0 - 1)/\gamma_0] d\ln p$ along constant values of m_0. Since the enthalpy is only weakly dependent on the pressure in this case it is not necessary to know the pressure distribution in the explosive zone exactly (for example, for a spherical explosion the formula $p/p_R = 0.4 + 0.6m/M$ gives a good approximation within a wide range of values of γ and $p_R/p_\infty > 1.5$). To solve the problem the initial profile $i(m/M)$ satisfying the energy balance must be given for some sufficiently small value of the mass M_{01}. The effect of this profile is damped out as the ratio M_0/M_{01} increases.

An example of this type of calculation for an explosion on the ground is given in Fig. 2 (solid line). We can see that μ has a minimum in the range of shock wave intensities corresponding to the maximum values of the function z in Fig. 1 in the main part of the shock wave. The asymptotic value $\mu \approx 0.78$ as $p_R \to p_\infty$ differs from unity, and this is is explained by the conservation of high temperatures in the central region (for a nonthermally conducting gas).

The coefficient μ given by the dashed line in Fig. 2 was obtained by processing the results given in paper [9] for the exact numerical calculations, and is fairly close to the approximate value. The quantity μ is determined by the region of values of m_0, corresponding to intense shock waves $f_1 \approx 1$, and since the constant $\varkappa_\nu$ is only weakly dependent on ν, the coefficient μ may be taken to be independent of the dimensionality of the space in the first approximation. The dash-dot line in Fig. 2 confirms this, since it gives data from paper [10] for $\nu = 2$ and is close to the curves for which $\nu = 3$.

We note that the form of representing the data in papers [9,10] excludes the possibility of using them with an accuracy greater than the discrepancy of the curves in Fig. 2.

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